SPECT3D Benchmark Calculation:
Radiative Transfer in 2-D Cylindrical R-Z Geometry

This memo describes benchmark calculations for 2-D multi-angle radiative transfer models used in SPECT3D. A multi-angle radiative transfer model for 2-D cylindrical R-Z geometry has recently been added to SPECT3D. This complements existing multi-angle models for 1-D planar, cylindrical, and spherical geometries. Below, we compare SPECT3D results with exact solutions.

In the calculations below, we assume the plasma to be spatially uniform. All calculations are done at a single frequency point. The size of the plasma is specified by the optical depth that corresponds to the radius of the cylinder, τ_R, and the optical depth that corresponds to the height of the cylinder, τ_Z (see Figure 1).

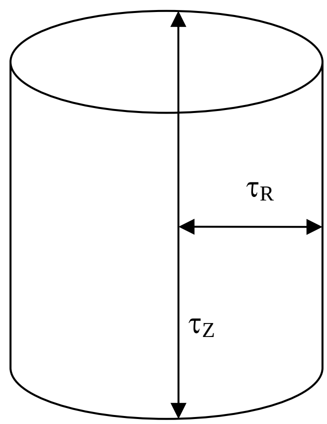

Figure 1. Schematic illustration of 2-D cylindrical R-Z geometry with size characterized by optical depths τ_R and τ_Z.

The photoionization and photoexcitation rates at a given location within a plasma depend on the mean intensity at that point. The mean intensity, J_ν, at a point *r* in a spatial grid is given by the angle average of the specific intensity I_ν:

$$J_\nu(r) = \frac{1}{4\pi} \int_{-1}^{1} d\mu \int_{0}^{2\pi} d\varphi \; I_\nu(r,\mu,\varphi) \tag{1}$$

where μ is the cosine of the polar angle, and φ is that azimuthal angle.

In cylindrical geometry, with a spatially uniform source function, S_ν, the exact solution is:

$$\frac{J_\nu}{S_\nu} = \frac{1}{4\pi} \int_{0}^{2\pi} d\varphi \int_{0}^{1} d\mu \left[2 - e^{-T_A(\mu,\varphi)} - e^{-T_B(\mu,\varphi)} \right] \tag{2}$$

where:

$$T_A(\mu,\varphi) = Min\left\{ T_A^R(\mu,\varphi), T_A^Z(\mu) \right\} \quad , \tag{3}$$

$$T_B(\mu,\varphi) = Min\left\{ T_B^R(\mu,\varphi), T_B^Z(\mu) \right\} \quad , \tag{4}$$

$$T_A^R(\mu,\varphi) = \frac{\tau_R}{\sqrt{1-\mu^2}} \left\{ \sqrt{1-\gamma_R^2 \sin^2(\varphi)} \; + \; \gamma_R \cos(\varphi) \right\} \; , \tag{5}$$

$$T_B^R(\mu,\varphi) = \frac{\tau_R}{\sqrt{1-\mu^2}} \left\{ \sqrt{1-\gamma_R^2 \sin^2(\varphi)} \; - \; \gamma_R \cos(\varphi) \right\} \; , \tag{6}$$

$$T_A^Z(\mu) = \frac{\tau_Z}{2\mu} \left(1-\gamma_Z\right) \; , \tag{7}$$

$$T_B^Z(\mu) = \frac{\tau_Z}{2\mu} \left(1+\gamma_Z\right) \; , \tag{8}$$

where γ_R is the scaled radius ($= r/R = \tau_r/\tau_R$), γ_Z is the scaled height ($= z/H = \tau_z/\tau_Z$), R is the radius of the cylinder, and H is the height of the cylinder.

Comparisons between SPECT3D results and exact solutions for 2-D cylindrical R-Z geometry are shown in Figures 2 and 3. In the SPECT3D calculations, we used a multi-angle discrete ordinates S_6 grid, in which each volume element in the grid sees radiation from a total of 24 angles. Figures 2 and 3 show the radial dependence of the mean intensity at several heights in the cylinder: $2z/H$ = 0.05 (*i.e.*, near the mid-plane of the cylinder), 0.45, 0.75, and 0.95 (near the top of the cylinder). Figure 2 shows results for τ_R = 1.825 and $\tau_Z = 2\,\tau_R$, while Figure 3 shows results for τ_R = 0.165 and $\tau_Z = 2\,\tau_R$. In each case, the agreement between the SPECT3D results and the exact solution is good, with typical differences being ~ a few percent. The differences are due to the finite number of angles in the SPECT3D angular grid.

In addition to the results shown, test calculations were performed to ensure the 2-D SPECT3D results were giving the appropriate solution for several limiting cases. These include: (1) large optical depth cases, where $J_\nu / S_\nu \to 1$ in the interior of the plasma; (2) a flattened cylinder with $H/R = \tau_Z/\tau_R \ll 1$, where the *z*-dependence of the mean intensity at *r* = 0 approaches that of the 1-D planar case; (3) a very tall cylinder with $H/R = \tau_Z/\tau_R \gg 1$, where the radial dependence of the mean intensity at *z* = 0 approaches that of the 1-D cylindrical case; and (4) a spherical plasma set up using a 2-D cylindrical R-Z mesh, where the solution is that of a 1-D spherical plasma. In all cases, the results agreed with those expected.

Uses of 2-D Modeling:

With the new 2-D radiative transfer modeling in SPECT3D, users will be able to model photoexcitation and photoionization processes in 2-D plasmas. One example is the post-processing of 2-D radiation-hydrodynamics calculations of capsule implosions to generate spectra that can be compared with experimental measurements. Another possible use is for experimentalists to set up hypothetical 2-D plasma distributions for dynamic hohlraums, and study physical processes that affect images and spectra obtained in experiments.

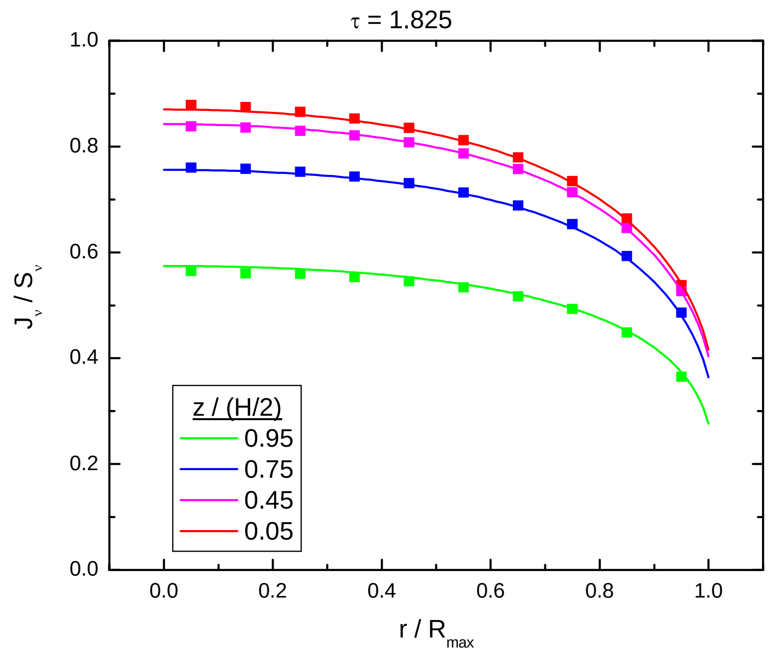

Figure 2. Spatial variation in mean intensity for a plasma on a 2-D cylindrical R-Z grid calculated using multi-angle (S_6) radiative transfer model in SPECT3D. Results are shown for several heights within the cylinder. The radius and height of the plasma are characterized by an optical depth of τ_R = 1.825 and τ_Z = 3.650. The SPECT3D results (symbols) are compared with exact solutions (solid curves).

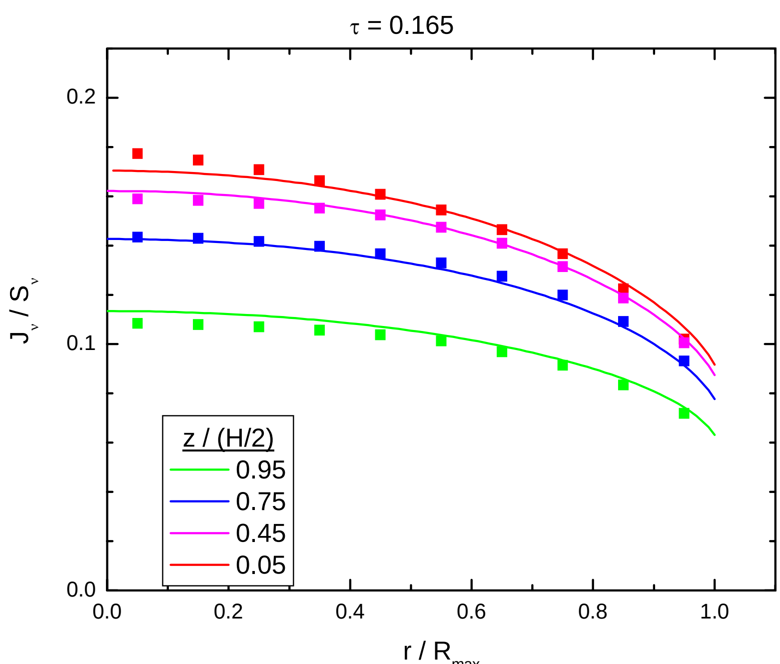

Figure 3. Same as Fig. 2, but with the radius and height of the plasma characterized by an optical depth of τ_R = 0.165 and τ_Z = 0.330.